Prof. Hadley

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THE

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NOVEMBER, 1860.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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Editorial Items.

Prizes for Vol. III. offered to all Students.

For Solutions. — A 1st Prize, $7.50; 2d, $5; 3d, $3; 4th, Vol. III. The third and fourth prizes are confined to beginners for solutions of Problems I. and II.; but as the best solutions only will be published, advanced students are desired to send solutions of all the problems.

For Essays. — A 1st Prize, $50; 2d, $40; 3d, $30; 4th, $20; 5th, $10. The first and second of these prizes are open to all competitors. Instead of cash prizes, we propose to make them all payable in mathematical books. This change will make the prizes less burdensome to us while it will be appropriate, and we believe quite as acceptable to competitors, to receive standard mathematical books, — such as Bowditch's edition of *Mécanique Céleste*, Peirce's *Analytic Mechanics*, Bartlett's *Analytic Mechanics*, Davis's edition of *Theoria Motus*, Davies and Peck's *Mathematical Dictionary*, Chauvenet's *Analytic Trigonometry*. But selections may be made from Sever & Francis's list, and desired works not found there will be ordered. To those preferring standard works in other departments, we offer Worcester's *Quarto Dictionary* and the fine editions of Hume, Gibbon, Lamb, and Macaulay, described in our advertising pages. To this list we may hereafter add editions of other standard authors, if desired.

A Generous Offer.

It gives us great pleasure to announce that "A Friend of the Mathematical Monthly Prizes," whose name we are not permitted to give, wishes to assume the responsibility of the Prizes for the solutions of the Problems in the number for December. He says, "The Monthly has been of great value to me. It has been my tutor, and certainly the tuition is cheap. I ought to be able now to contribute something more to its sustenance." The solutions will be due February 1, 1861, and the Report will be published in the February number. We hope to receive a large number of solutions as evidence of an exciting contest for these Prizes. It will be seen (Editorial Items), that the times for the solutions of the September and October Problems are extended one month. Hereafter we shall issue the Monthly promptly on or before the first of the month.

Portrait of Sir John Herschel.

Through the kindness of Miss Maria Mitchell we have received a fine photograph of Sir John Herschel, which we shall have engraved on steel for an early number.

We expect our friends will aid us by increasing our subscription list. See Mathematical Monthly, on seventh page of Advertiser.

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Supplemens au traité de mécanique céleste

Depuis l'impression du traité de mécanique céleste, j'ai publié sur les divers objets de cet ouvrage, un grand nombre de recherches insérées dans les mémoires de l'institut et de l'académie des Sciences et dans la volumes de la Connoissance des tems. je vais ici réunir ces recherches dans l'ordre qui me semble le plus convenable, en leur donnant toute la généralité dont elles m'ont paru susceptibles. je vais commencer par ce qui concerne la figure des corps célestes, et les oscillations des fluides qui les recouvrent.

Autographe de Laplace.

Cette note, écrite de la main du célèbre géomètre française a été donnée à Mr. Watson par Madame la Marquise de Laplace sa veuve, qui en a certifié en ma présence la parfaite authenticité.

Ch. De Saussay

membre de l'institut de France.

THE

MATHEMATICAL MONTHLY.

Vol. III. . . . NOVEMBER, 1860. . . . No. II.

PRIZE PROBLEMS FOR STUDENTS.

I. Prove that the surface of a sphere can be completely covered with the surfaces either of 4, or of 8, or of 20 equilateral spherical triangles.

II. Which is greater, $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+$ &c., *ad inf.*, or $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+$ &c., *ad inf.*?

III. $$\frac{(x+a+b)^5+(x+c+d)^5}{(x+a+c)^5+(x+b+d)^5}=\frac{m}{n}.$$

Find x by quadratics.

IV. A fraudulent tradesman contrives to employ his *false* balance both in buying and selling a certain article, thereby gaining at the rate of 11 per cent more on his outlay than he would gain if the balance were *true*. If however the scale-pans, in which the article is weighed when bought and sold respectively, were interchanged, he would neither gain nor lose by the article. Find the legitimate gain per cent on the article.

V. Show that in the parabola the half sum of the radii vectores drawn to the extremities of any arc whatever, is equal to the radius vector drawn to the summit of the diameter passing through the centre of the chord and parallel to the axis, plus the part of this diameter intercepted between the arc and the chord.— Communicated by Asaph Hall, Esq.

Solutions of these problems must be received by January 1, 1860.

REMARKS ON THE PROBLEM OF THE LIGHTS.*

By Prof. W. H. Parker, Middlebury College, Vermont.

I am very much obliged to you for showing, in your editorial remarks on my "Note" in the February number, page 160, Vol. I., that the error which I conceived to be one of interpretation, really consists in a fallacy, to which the misinterpretation is only incidental.

Having re-examined the subject from a new point of view, suggested by your remarks, and with particular reference to the question, how can we determine from our equations whether the conditions of a problem are impossible or not? I desire, with your permission, to add the following remarks.

The general equation of our problem is

$$\text{(A)} \qquad \frac{a}{x^2} = \frac{b}{(c-x)^2},$$

whose roots are $\frac{c\sqrt{a}}{\sqrt{a} \pm \sqrt{b}}$. If in the roots we suppose $c = 0$, both values of $x = 0$. In order to determine whether we have here the true roots of our equation, under the particular supposition, let us go back to equation (A), and change it to the form which will express the condition $c = 0$. We have now

$$\text{(B)} \qquad \frac{a}{x^2} = \frac{b}{x^2},$$

the same form which we should have obtained by stating the particular problem which this enunciates. Our roots 0 do not satisfy this equation. A moment's consideration is sufficient to show why they should not. So long as a and b are unequal, the expression

* These Remarks reply to some incidental points in our article, the correctness of the main question being admitted. It is proper to state that this article was received shortly after the publication to which it refers, and has been delayed for want of space. — Ed.

$\frac{a}{x^2} = \frac{b}{x^2}$ is not a true equation; for we have unequals a and b divided by the same quantity, x^2, and no value of x (not even $x = 0$) will make the members equal. This shows conclusively the impossibility of the conditions. When, therefore, our problem requires us to find the point or points of equal illumination of two unequal lights, which are placed together, it requires what the algebraic enunciation of the conditions shows to be impossible.

In like manner, if we take the form of the problem in the editorial remarks, whose equation is

$$\text{(C)} \qquad \frac{a}{(x-\alpha)^2} = \frac{b}{(\beta-x)^2},$$

and whose roots are $\frac{\beta\sqrt{a} \pm \alpha\sqrt{b}}{\sqrt{a} \pm \sqrt{b}}$; and in the roots suppose $\beta = \alpha$, we obtain $x = \alpha$. To determine whether α is a true root, let us go back to equation (C), from which it was derived. Making $\beta = a$, we get

$$\text{(D)} \qquad \frac{a}{(x-\alpha)^2} = \frac{b}{(a-x)^2},$$

a form of equation which α will not satisfy. Hence α is not one of the roots. Indeed the expression $\frac{a}{(x-\alpha)^2} = \frac{b}{(a-x)^2}$ has no roots, for the reason that it is not an equation. For here, too, we have unequals a and b divided by equals; and here, just as in the former case, we find the conditions to be impossible, because we see that the expressions for the intensities cannot be equal.

With the knowledge that the expressions $\frac{a}{x^2} = \frac{b}{x^2}$ and $\frac{a}{(x-\alpha)^2} = \frac{b}{(a-x)^2}$ are not equations at all, it is absurd for us to attempt their reduction, or to ask for their roots. From false premises, surely, we cannot expect reliable conclusions.

I deem it important to remark here, that, since 0 is not a true root of equation (B), nor α of equation (D), we have in neither of them the symbol of infinity. How, then, is the impossibility of the conditions made known? Simply, as it seems to me, by the untruth-

fulness of the equations; or, in other words, by the impossibility of forming the conditions into real equations.

The idea that an impossibility in the conditions will always manifest itself somewhere by the symbol of infinity, I cannot think to be well founded. In the cases already considered, we have an impossibility without the symbol of infinity. On the other hand, we may have the symbol, in a true equation, without any impossibility in the conditions. This is made apparent by considering the case in which $a = b$ and $c = 0$ in equation (A), producing the form $\frac{a}{x^2} = \frac{a}{x^2}$. This is an identical equation, and of course true for any value of x, including $x = 0$. When $x = 0$, we have in each member the symbol of infinity, though we have no impossibility in the conditions. For when the lights are together, and of equal intensity, the point where they are placed is truly a point of equal illumination.

If in equation (C) we make $a = b$ and $\beta = \alpha$, it comes $\frac{a}{(x-\alpha)^2} = \frac{a}{(\alpha - x)^2}$, which is also an identical equation, and, as before, a case of real indetermination. The equation is true for any value of x, including $x = \alpha$. But when $x = \alpha$, each member is a symbol of infinity; and yet here we have no impossibility, because when the equal lights are placed together at the distance α from the origin, the point they occupy is equally illuminated by them.

It will be observed that I have not been speaking of the symbol of infinity in the *roots* of an equation.

The error of Bourdon seems to me to have originated in his failing to observe that the supposition $c = 0$, in the equation of the problem, destroys the equality of the members, and makes his premises false. Reasoning from them as if true, which is his first error, he falls into another at the point indicated in the editorial remarks, page 179; not precisely because, in any form of the problem, the particular *hypothesis* introduces a factor *zero* (since α is not a true root of (D), nor 0 of (B)), but because, in reducing the expression

(B) as if it were an equation, he obtains a form ($a x^2 = b x^2$, or $(a - b) x^2 = 0$) whose members would be equal, if $x = 0$, and hence infers that x is equal to 0 in form (B); thus surreptitiously admitting the factor *zero*, rather than legitimately introducing it by the particular hypothesis.

In a true equation, I do not discover that the presence of *zero* as a factor leads to any fallacy, so long as the factor remains. The removal of a factor *zero* from a real equation may introduce a fallacy, but it is one which the algebraist is apt to discover if he has inadvertently fallen into it.

One word in reference to a seeming antagonism between my remarks on the reliability of the algebraic processes as a method of reasoning, and the editorial remarks in regard to the "*implicit fallacy* to which algebraic transformations are often liable." The difference I suppose to be rather apparent than real. I have been accustomed to regard the fallacies which sometimes occur in our use of the algebraic methods as resulting from errors in ourselves. When our first equation is true, and our transformations are legitimately made, i. e. in strict accordance with those necessary truths which underlie all changes of form in equations, I have supposed we might always rely implicitly upon our conclusions. In this confidence I can hardly think myself singular.

If it be true, that there are no fallacies inherent in the algebraic processes, is it not better to speak of the fallacies to which *we* are often liable in algebraic transformations, than to use a form of expression which seems inconsistent with the absolute integrity of those processes as a method of reasoning?

The above remarks are submitted, in the hope that, if they fail to illustrate the topics discussed, they will at least furnish the occasion for remarks from yourself or others, which shall bring the truth to light.

NOTE ON THE PROBLEM OF THE LIGHTS.

By Hugh Godfray, St. John's College, Cambridge, England.

May I be allowed to make a few remarks on the difficulty in the interpretation of the Problem of the Two Lights, noticed by Professor Parker in Vol. I., page 160, of the Monthly?

Let us consider the problem somewhat differently, and suppose the two lights together at the outset.

D' Q' O P Q D

Let two lights, A, B, of intensities a, b, at unit of distance, be placed together at O in the line $D'D$, and let a point, P, which receives light from A, move from D through O towards D'; we can find the corresponding positions of a point, Q, which moves in such a way as to receive from B the same illumination as P does from A. (We suppose b to have a finite ratio to a.)

The equation connecting the distances will be

$$\frac{a}{OP^2} = \frac{b}{OQ^2};$$

whence

$$OQ = \pm OP\sqrt{\frac{b}{a}},$$

which shows that there are two positions, Q, Q', at equal distances to the right and left of O corresponding to each position of P.

Let us confine our attention to the $+$ sign, which gives that position of Q which is on the same side of O as P.

$$OQ = OP\sqrt{\frac{b}{a}}.$$ *

* If we suppose that OQ and OP are both absolute *zeros*, that is, that the two lights occupy absolutely the same point, then the equation $OQ = OP\sqrt{\frac{b}{a}}$ is not true, because $\frac{OQ}{OP} = 1$, and not the constant ratio of $\sqrt{\frac{b}{a}}$. A light of a fixed intensity at a unit's distance must illuminate

We see that $O\,Q$ is in a constant ratio to $O\,P$. When $O\,P$ diminishes, $O\,Q$ diminishes; when $O\,P$ becomes negative, or P passes to the left of O, $O\,Q$ also becomes negative, and Q passes to the left of O, the two points P and Q passing through O simultaneously, a result which agrees with that of the particular case in the usual way of considering the problem; but perhaps the above method of arriving at it may tend to explain away the difficulty experienced in the other.

It is true that when the lights are of unequal intensity there is no point on either side of O equally illuminated; for if $O\,P$ has any finite value, $O\,Q$ has a different value; but the distance between these two points, which is

$$O\,P\,\frac{\sqrt{b}-\sqrt{a}}{\sqrt{a}},$$

diminishes with $O\,P$, and vanishes when $O\,P$ vanishes, whatever be the relative intensities of A and B.

The value *zero*, therefore, in the usual problem is neither superfluous nor erroneous. The apparent contradiction involved in the statement, that if we admit this value we must also admit that the point O is at the same time equally and unequally illuminated, will disappear if we consider that when the point P is indefinitely near to O, the point Q, which is equally illuminated, is also indefinitely near to O; and points at which the illumination from B is 2, 3, 4, &c. times that at P will at the same time, as can be shown in a similar manner, be indefinitely near to O, all these points coinciding with O at the same instant that P does.

---

the point it occupies by a fixed amount, if it illuminates this point at all; and unless we can arbitrarily change the intensity, without producing a corresponding change in the amount by which this point is illuminated, the ordinary solution of the problem for the special case when the two lights are together must be untrue.

We have already shown (page 178, Vol. I.), that, under the hypothesis of the lights occupying the same point, the problem fails altogether, it being impossible to satisfy the corresponding equation with *zero*, or any other value of x. — Ed.

The problem as solved by BOURDON and others would therefore be incomplete without the value *zero*, which I consider a perfectly legitimate solution of the particular case.

The very fact of putting a problem into algebraic language invests it with all the generality the symbols we introduce admit of, and the results of our investigations are nearly always more general and complete than we anticipated in the particular question we proposed to ourselves; and if difficulties of interpretation or superfluous solutions arise, they will be found to have been introduced in the process of reduction, or to be due to the impossibility of expressing in the equation all the restrictions and limitations of the problem, which is not the case in the present instance.

AN EASY MODE OF APPROXIMATING TO THE TIME OF VIBRATION IN A CIRCULAR ARC.

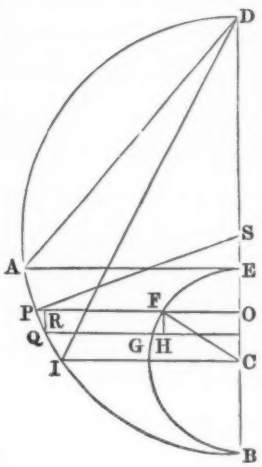
By PROF. LEWIS R. GIBBES, College of Charleston, South Carolina.

LET a particle descend a circular arc AB, whose centre is S, urged by a uniformly accelerating force, acting always parallel to DB, the vertical diameter of the circle; it is required to find T, the time of descent, the arc being supposed not more than a quadrant.

Through A, the point from which the descent begins, draw the horizontal line AE; on EB the versed sine of the arc of descent AB as a diameter, describe the semi-circle EFB, whose centre is C; let PQ be an element of the arc AB, and draw the other lines as shown in the figure, whose construction will be easily understood.

Suppose the particle descending from A to have reached the point P, and about to describe the element PQ in the next element of time, with the acquired velocity continued uniformly; put f to express the intensity of the accelerating force, or the additional velocity

acquired in each second of time; also r for the radius SP or SB, or length of the simple pendulum vibrating in the given arc; $\sqrt{(2f \times EO)}$ will express the velocity acquired in descending AP, with which velocity the element PQ is described.



From the elementary triangle PQR, and the similar one SPO, we have

$$PQ \div QR = SP \div PO;$$

from the elementary triangle FGH, and the similar one FCO, we have

$$FH \div FO = FG \div FC;$$

by properties of the circle, we have

$$FO \div \sqrt{(EO \times OB)} = PO \div \sqrt{(DO \times OB)};$$

also write the identity $\sqrt{2f} = \sqrt{2f}$;

and the equality $SP = \sqrt{(\frac{1}{2}DB \times SB)}$.

Hence, observing that $FH = QR$, we obtain

$$\frac{PQ}{\sqrt{(2f \times EO)}} = \frac{FG}{FC} \times \sqrt{\frac{\frac{1}{2}SB}{2f}} \times \sqrt{\frac{DB}{DO}}.$$

This may be written

$$\frac{\text{elementary arc } PQ}{\sqrt{(2f \times EO)}} = \frac{\text{elementary arc } FG}{2FC} \times \sqrt{\frac{r}{f}} \times \sqrt{\frac{DB}{DO}}.$$

Now, since $\frac{\text{elementary arc } PQ}{\sqrt{(2f \times EO)}} = \frac{\text{space}}{\text{velocity}}$ = the element of time in which is described the elementary space PQ, if $\sqrt{\frac{DB}{DO}}$ were a constant, we could write, the sum of the elements of time employed in descending the successive elements of space of which PQ is one =

$$\frac{\text{sum of the elementary circular arcs of which } FG \text{ is one}}{\text{diameter of the circle}} \times \sqrt{\frac{r}{f}} \times \sqrt{\frac{DB}{DO}},$$

and since the sum of the elements of time constitutes T, the whole time of descent down AB, and the sum of the elementary circular arcs, of which FG is one, constitutes the semi-circumference EFB, and $\frac{\text{semi-circumference}}{\text{diameter}}$ is equal to $\frac{\pi}{2}$, we might write

$$T = \tfrac{\pi}{2}\sqrt{\tfrac{r}{f}} \times \sqrt{\tfrac{DB}{DO}}.$$

But $\sqrt{\frac{DB}{DO}}$ has a different value for every such elementary arc as FG, and we require, not the sum of the elementary arcs, but the sum of the products of each elementary arc by the corresponding value of $\sqrt{\frac{DB}{DO}}$; this summation constitutes the difficulty of the problem, and may be effected by the use of arcs of the ellipse and hyperbola. In its place we will substitute an approximation by using a *mean value* of $\sqrt{\frac{DB}{DO}}$.

The greatest value of $\sqrt{\frac{DB}{DO}}$ is $\sqrt{\frac{DB}{DE}}$, and the least is $\sqrt{\frac{DB}{DB}} = 1$, the arithmetical mean of these values is $\frac{1}{2}\left(1+\sqrt{\frac{DB}{DE}}\right)$, which, substituted in the equation for T, gives

$$T = \tfrac{\pi}{2}\sqrt{\tfrac{r}{f}} \times \tfrac{1}{2}\left(1 + \sqrt{\tfrac{DB}{DE}}\right).$$

By the properties of the circle, $\sqrt{\frac{DB}{DE}} = \frac{DA}{DE}$, and this ratio is the secant of the angle ADB, or the secant of half the arc AB; if this arc AB, the arc of excursion of the pendulum, be A, then

$$T = \tfrac{\pi}{2}\sqrt{\tfrac{r}{f}} \times \frac{1+\sec\frac{1}{2}A}{2},$$

as an approximate value for the time. The subjoined table gives, for every 10° of arc A in the first column, the corresponding value of $\frac{1+\sec\frac{1}{2}A}{2}$, in the second column.

Another mode of approximation presents itself. The greatest

value of DO is DB, and the least is DE, and we may take as its mean value DC, which corresponds to the centre of the circle described on EB as a diameter, and substitution gives

$$T = \frac{\pi}{2}\sqrt{\frac{r}{f}} \times \sqrt{\frac{DB}{DO}};$$

A.	$\frac{1+\sec \frac{1}{2}A}{2}$	a.	$\sec \frac{1}{2}a$.	Mean of 2d & 4th Columns.
°		° ′		
10	1.001910	7.32	1.001904	1.001907
20	1.007713	14.06	1.007618	1.007666
30	1.017638	21.05	1.017168	1.017813
40	1.032089	28.00	1.030614	1.031351
50	1.050689	34.47	1.047954	1.049822
60	1.077350	41.25	1.069011	1.073181
70	1.110387	47.51	1.093929	1.102159
80	1.152704	54.04	1.122659	1.137681
90	1.207107	60.00	1.154701	1.180904

as $\frac{DB}{DO} = \frac{DI}{DO}$ = secant of IDC, or the secant of half the arc IB, if this arc be expressed by a, we shall have $T = \frac{\pi}{2}\sqrt{\frac{r}{f}} \times \sec \frac{1}{2}a$. The arc a may be found from A, the given arc of excursion by either of the following equations: —

$$\textit{versed sine } a = \tfrac{1}{2} \textit{ versed sine } A; \qquad \cos a = \cos^2 \tfrac{1}{2} A;$$
$$\sin \tfrac{1}{2} a = \sin \tfrac{1}{2} A \sin 45°.$$

If EB, the height of the arc of descent AB, be expressed by h, then $\textit{versed sine } a = \frac{h}{2r}$. The table gives, for every 10° of arc A in the first column, the corresponding value of arc a in the third column, and of sec $\frac{1}{2}a$ in the fourth.

As there is nothing in the investigation to indicate which method will give the closest approximation to the truth, we may take the arithmetical mean of the two as a final result, and we obtain

$$T = \frac{\pi}{2}\sqrt{\frac{r}{f}} \times \frac{1+\sec \frac{1}{2}A + 2\sec \frac{1}{2}a}{4},$$

as a final approximate expression for the time of descent in a circular arc of A degrees, A being not more than 90°. The fifth column of the table gives the mean of the second and fourth columns, which corresponds to the last factor in the preceding expression. As neither of the two approximations differs from the mean by half a unit in the fifth decimal place when $A = 10°$, either may be used for cal-

culating T when A is less than 10°; when A is 5°, neither will differ from the mean in the sixth decimal place. The first will be the most convenient expression when A is given; the second when h is given.

The double of either of the three expressions for T will of course be the time of one complete vibration of the pendulum, whose length is r and angle of excursion A. When A is very small, the last factor in each of the above expressions will differ very little from unity, and we may then put $T = \frac{\pi}{2}\sqrt{\frac{r}{f}}$, the double of which is the time of vibration in the arc of a cycloid whose axis is vertical and equal to $\frac{r}{2}$. The geometrical construction used above can easily be applied to the cycloid, to deduce this result.

It may, perhaps, be assumed with safety, that, for small values of A, our final result does not differ from the truth more than the two first approximations do from each other; but, for large values of A, its error can only be determined by comparing it with the true value of T, furnished by the analytic treatment of the problem. As this value is expressed by $\frac{\pi}{2}\sqrt{\frac{r}{f}}$ multiplied by a certain series, we must expand the last factor in each of our expressions above into a series, and then we need only compare the series together, as the quantity $\frac{\pi}{2}\sqrt{\frac{r}{f}}$ is common to all the expressions. Since DB is $2r$, $DE = 2r - h$, $DC = 2r - \frac{1}{2}h$, in the first mode of approximation,

$$\tfrac{1}{2}\left(1+\sqrt{\frac{DB}{DE}}\right) = \tfrac{1}{2}\left(1+\sqrt{\frac{2r}{2r-h}}\right) = \tfrac{1}{2}\left(1+\left(1-\frac{h}{2r}\right)^{-1}\right),$$

and in the second mode,

$$\frac{DB}{DC} = \sqrt{\frac{2r}{2r-\frac{1}{2}h}} = \left(1-\tfrac{1}{2}\cdot\frac{h}{2r}\right)^{-1};$$

developing these expressions by the binomial theorem, we shall have

the first and second series given below (the third line contains the arithmetical mean of the two), which series is rewritten on the fourth line, slightly altered in form for readier comparison with the true series, which is written on the fifth line. In the general term, n indicates the order, beginning with the second.

$$1+\tfrac{1}{2}\cdot\tfrac{1}{2}\left(\tfrac{h}{2r}\right)+\tfrac{1}{2}\cdot\frac{1.3}{2.4}\left(\tfrac{h}{2r}\right)^2+\tfrac{1}{2}\cdot\frac{1.3.5}{2.4.6}\left(\tfrac{h}{2r}\right)^3+\tfrac{1}{2}\cdot\frac{1.3.5.7}{2.4.6.8}\left(\tfrac{h}{2r}\right)^4+\text{\&c.}$$

$$1+\tfrac{1}{2}\cdot\tfrac{1}{2}\left(\tfrac{h}{2r}\right)+\frac{1}{2^2}\cdot\frac{1.3}{2.4}\left(\tfrac{h}{2r}\right)^2+\frac{1}{2^3}\cdot\frac{1.3.5}{2.4.6}\left(\tfrac{h}{2r}\right)^3+\frac{1}{2^4}\cdot\frac{1.3.5.7}{2.4.6.8}\left(\tfrac{h}{2r}\right)^4+\text{\&c.}$$

$$1+\tfrac{1}{2}\cdot\tfrac{1}{2}\left(\tfrac{h}{2r}\right)+\tfrac{1}{2}\cdot\frac{2+1}{2^2}\cdot\frac{1.3}{2.4}\left(\tfrac{h}{2r}\right)^2+\tfrac{1}{2}\cdot\frac{2^2+1}{2^3}\cdot\frac{1.3.5}{2.4.6}\left(\tfrac{h}{2r}\right)^3+\tfrac{1}{2}\cdot\frac{2^3+1}{2^4}\cdot\frac{1.3.5.7}{2.4.6.8}\left(\tfrac{h}{2r}\right)^4+\text{\&c.}$$

$$1+\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\left(\tfrac{h}{2r}\right)+\frac{1.3}{2.4}\cdot\frac{1.3}{2.4}\left(\tfrac{h}{2r}\right)^2+\frac{1.3.5}{2.4.6}\cdot\frac{1.3.5}{2.4.6}\left(\tfrac{h}{2r}\right)^3+\frac{1.3.5.7\frac{1}{2}}{2.4.6.8}\cdot\frac{1.3.5.7}{2.4.6.8}\left(\tfrac{h}{2r}\right)^4+\text{\&c.}$$

$$1+\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\left(\tfrac{h}{2r}\right)+\frac{1.3}{2.4}\cdot\frac{1.3}{2.4}\cdot\left(\tfrac{h}{2r}\right)^2+\frac{1.3.5}{2.4.6}\cdot\frac{1.3.5}{2.4.6}\cdot\left(\tfrac{h}{2r}\right)^3+\frac{1.3.5.7}{2.4.6.8}\cdot\frac{1.3.5.7}{2.4.6.8}\left(\tfrac{h}{2r}\right)^4+\text{\&c.}$$

On comparing the last two lines, it will be seen that the series furnished by our third or final approximation does not diverge from that furnished by analysis, until the term containing the *fourth power* of the variable $\frac{h}{r}$ the versed sine of A, and errs in excess; yet for $A = 60°$, it gives a result *true to the sixth decimal place*, and, even for $A = 90°$, gives an excess of only half a unit in the third decimal place; that is, of only one five-hundredth of the whole. It is therefore abundantly sufficient for the determination of the variation in the rate of a clock depending on the variation in the extent of the arc of vibration.

The idea of using an approximation to avoid the difficulties of the problem is not new, nor is the geometric construction, adopted in the demonstration, presented as novel, although we are unable to say who first used it; but we have never seen it handled in such a manner as to secure the advantages that belong to the geometric method when the subject is within its scope. Sometimes the calculus is applied to the geometric construction, and then the indepen-

dence and comprehensiveness of the calculus is sacrificed in the combination. We have endeavored so to exhibit the subject, as to secure the simplicity and directness which are the characteristics of the geometric method, and our three modes of approximation, which seem very naturally to present themselves, are, as far as we are aware, new, — never having fallen in our way, — and, with the aid of the usual trigonometrical tables, furnish a far readier means of computing the time in a circular arc than does the series given by the calculus.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. XI., Vol. II.

The first Prize is awarded to Asher B. Evans, Madison Univ., N. Y.

The second prize is awarded to John A. Winebrener, Princeton College, N. J.

The third prize is awarded to E. O. Gibson, South New Berlin, N. Y.

Prize Solution of Problem I.

Prove that an equation of the second degree cannot have more than two roots.

The general equation of the second degree is

$$Ax^2 + Bx + C = 0.$$

If this equation can have more than two roots, say three, denote them by a, b, c; then

$$Aa^2 + Ba + C = 0, \quad Ab^2 + Bb + C = 0, \quad Ac^2 + Bc + C = 0.$$

By subtraction, and dividing by the factors $a - b$, $a - c$, $b - c$, which by hypothesis are not zero, we get

$$A(a+b) + B = 0, \quad A(a+c) + B = 0, \quad A(b+c) + B = 0.$$

By subtracting again,

$$A(b-c)=0,\quad A(a-c)=0,\quad A(a-b)=0.$$

But these equations are impossible, since A is not zero. Hence, the given quadratic cannot have three different roots.

Prize Solution of Problem II.

By E. O. Gibson, South New Berlin, New York.

The sum of n numbers in arithmetical progression is a, and the sum of their squares is b. Required, the first term and the common difference.

Let x denote the first term, and y the common difference; then

(1) $x+(x+y)+(x+2y)+\ldots.+(x+(n-1)y)=a.$

(2) $x^2+(x+y)^2+(x+2y)^2+\ldots.+(x+(n-1)y)^2=b;$

or, by reduction,

(3) $$nx+\tfrac{1}{2}n(n-1)y=a,$$

(4) $$nx^2+n(n-1)xy+\tfrac{1}{6}n(n-1)(2n-1)y^2=b.$$

From n times (4) subtract the square of (3); we get

$$\frac{n^2(n^2-1)}{12}y^2=bn-a^2.\quad\therefore y=\pm\frac{2}{n}\sqrt{\left(\frac{3(bn-a^2)}{n^2-1}\right)}.$$

By substituting this value of y in (3), we find

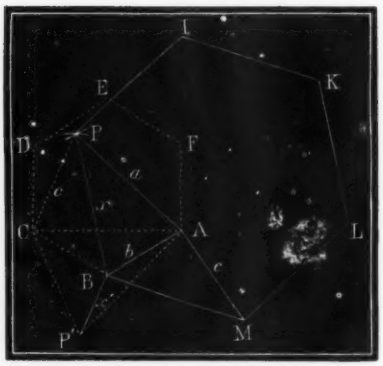
$$x=\frac{a}{n}\pm\frac{1}{n}\sqrt{\left(\frac{3(n-1)(bn-a^2)}{n+1}\right)}.$$

Prize Solution of Problem III.

Given, three lines, a, b, c, drawn from any point within a regular polygon of n sides to any three of its consecutive corners, A, B, C. Required, the side of the polygon, and also a geometrical solution, granting that a polygon of n sides can be constructed when one side is given.

Upon the line AB, taken equal to (b), construct the polygon of (n) sides, $ABCDEF$ (in the figure $n=6$), and join two alternate corners, as A and C, by the diagonal AC. From A as a centre,

with a radius equal to (a), describe an arc, and taking C as a centre, with a radius equal to (c) describe another arc, cutting the first at P, taking care that P and B line on opposite sides of the line $A\,C$. Join $P\,B$, and upon it describe the polygon $B\,P\,I\,K\,L\,M$, which is the one required. For, by construction, $A\,P = a$ and $A\,B = b$. Also, the angle $C\,B\,A = P\,B\,M$, since they are angles of regular polygons of the same number of sides. Subtract $P\,B\,A$, and $C\,B\,P = A\,B\,M$; also, by construction, $C\,B = A\,B$ and $P\,B = B\,M$. $\therefore A\,M = C\,P = c$.



Schol. — If P lie on the *same* side with B, of the line $A\,C$ (as at P'), then will $B\,P'$ be the side of another polygon of n sides, which will fulfil all the conditions of the problem, except that the point A, from which the three lines are drawn, lies *outside* of it.

This solution, by Prof. C. A. Young, Western Reserve College, Hudson, Ohio, is neater than any received, and therefore we give it.

Prize Solution of Problem IV.

By all the Competitors.

Deduce the formulæ of Arithmetical and Geometrical Progressions bv the method of differences.

If d_1, d_2, d_3, &c. denote the successive orders of difference, a the first term and s the sum of n terms, then the usual formula is

$$(1) \qquad S = n\,a + \frac{n\,(n-1)}{1\,.\,2}\,d_1 + \frac{n\,(n-1)\,(n-2)}{1\,.\,2\,.\,3}\,d_2 + \text{\&c.}$$

In an arithmetical progression the first differences are constant, and therefore d_2, d_3, &c. equal zero; hence

$$S = na + \frac{n(n-1)}{1.2}d = \frac{n}{2}(a + a + (n-1)d) = \frac{n(a+b)}{2}.$$

In a geometrical progression,

$$S = a + ar + ar^2 + \ldots.$$

$$d_1 = a(r-1), \quad d_2 = a(r-1)^2, \quad d_3 = a(r-1)^3, \text{ \&c.}$$

Therefore (1) becomes

$$S = na + \frac{n(n-1)}{1.2}a(r-1) + \frac{n(n-1)(n-2)}{1.2.3}a(r-1)^2 + \text{\&c.}$$

$$= \frac{a}{r-1}\left\{n(r-1) + \frac{n(n-1)}{1.2}(r-1)^2 + \frac{n(n-1)(n-2)}{1.2.3}(r-1)^3 + \text{\&c.}\right\}$$

$$= \frac{a}{r-1}\left\{(r-1+1)^n - 1\right\} = \frac{a(r^n-1)}{r-1},$$

since the series in { } lacks 1 of being the nth power of $1 + (r-1)$.

Prize Solution of Problem V.

By John A. Winebrener, Princeton College, New Jersey.

If three points on the surface of a sphere (radius = 1) be joined by three arcs of small circles whose angular radii are ϱ, ϱ', ϱ'', the area of the included triangle will be

$$\theta + \theta' + \theta'' + \pi \pm \varphi \cot \varrho \pm \varphi' \cot \varrho' \pm \varphi'' \cot \varrho'';$$

in which φ, φ', φ'' are the lengths of these arcs, and θ, θ', θ'' the angles at which they intersect.

Let ABC be the given triangle. Take any point P within it, and join PA, PB, PC. Let O be the pole of BC, or φ, and join OB, OC. Since $\sin \varrho$ is the planar radius of φ, we have by geometry

$$(1) \quad \angle BOC : \varphi = 1 : \sin \varrho. \quad \therefore \angle BOC = \frac{\varphi}{\sin \varrho}$$

The surface of a zone, of which OBC is a part, equals $2\pi(1 - \cos \varrho)$. Therefore the surface of

$$(2) \qquad OBC = \frac{\varphi}{\sin \varrho}(1 - \cos \varrho).$$

Denote the angles of the triangle PBC by $\alpha, \alpha', \alpha''$ · then the surface of

$$(3) \qquad PBOCP = \alpha + \alpha' + \alpha'' - \pi - \frac{\varphi}{\sin \varrho},$$

since $\angle PBC = \alpha' - 90°$, &c.

Adding (2) and (3) the surface of

$$(4) \qquad PBC = \alpha + \alpha' + \alpha'' - \pi - \varphi \cot \varrho.$$

The triangles APB and APC will give similar results, and by adding these, and noting that the angles $A + B + C = \theta + \theta' + \theta''$, and the angles at $P = 2\pi$, we have finally the surface of

$$(5) \quad ABC = \theta + \theta' + \theta'' - \pi \pm \varphi \cot \varrho \pm \varphi' \cot \varrho' \pm \varphi'' \cot \varrho''.$$

Note by Asher B. Evans. — If $\theta_1, \theta_2, \ldots \theta_n$; $\varphi_1, \varphi_2, \ldots \varphi_n$; $\varrho_1, \varrho_2, \ldots \varrho^n$ represent similar quantities for a polygon formed on the surface of a sphere (radius $= 1$), by joining n points in order, by arcs of small circles, then the area of this polygon will be

$$(6) \quad \theta_1 + \theta_2 + \ldots \theta_n - (n-2)\pi \pm \varphi_1 \cot \varrho_1 \pm \varphi_2 \cot \varrho_2 \pm \ldots \varphi_n \cot \varrho_n.$$

If $n = 3$, (5) and (6) are identical; if $n = 2$, (6) becomes

$$\theta_1 + \theta_2 \pm \varphi_1 \cot \varrho_1 \pm \varphi_2 \cot \varrho_2 = 2\theta_1 \pm \varphi_1 \cot \varrho_1 \pm \varphi_2 \cot \varrho_2,$$

which is the area of a lune formed by the intersection of two small circles, whose radii are ϱ_1 and ϱ_2, angle of intersection θ_1, φ_1 and φ_2 denoting the lengths of the arcs.

Simon Newcomb,

W. P. G. Bartlett,

Truman Henry Safford.

NOTES AND QUERIES.

Note on the Greatest Common Measure.

I. The difference between two multiples of a number must be a multiple of that number.

II. Any times a multiple of a number must be a multiple of that number.

Admit the above, and the following solutions are easily uuderstood.

1. Find the greatest common measure of 6 and 28; 72 and 486; 252 and 462; 1573 and 3553; 1181, 2741, and 3413.

2. Mathematical Monthly, Vol. I. p. 5, Ex. 2. "Required the greatest common divisor of 1633, 3763, 4757, and 4189."

Solution of No. I.

6) 28 (5 — 72) 486 (7 — 252) 462 (2 — 1573) 3553 (2

30 — 504 — 504 — 3146

2 Ans. — 18 Ans. — 42 Ans. — 407) 1573 (4

1628

55 = 5.11.

Ans. 11.

1181) 2741 — 3413

2362 — 3543

379 — 130 = 10.13. Neither 2, 5, nor 13 measures 379.

∴ Ans. 1.

Solution of No. II.

1633) 3763 — 4757 — 4189

3266 — 4899 — 4899 — 1633

497 = 7.71 — 142 = 2.71 — 710 = 10.71 — 71 = 23.

Ans. 71.

The advantage, in many cases, of taking a quotient figure too large by unity, which may be done in any stage of the process, is readily seen. The products could be written over the dividend, if thought necessary. — E. Hunt, Sub-Master English High School, Boston.

Notes from Todhunter's Algebra.

I. *Determine the conditions which make the binomial* a $\pm \sqrt{}$b *a perfect cube.*

If we assume

(1) $\sqrt[3]{(a+\sqrt{b})} = x+\sqrt{y}$; then (2) $\sqrt[3]{(a-\sqrt{b})} = x-\sqrt{y}$.

For $a+\sqrt{b} = (x+\sqrt{y})^3 = x^3+3x^2\sqrt{y}+3xy+y\sqrt{y}$; and, since the rational parts, as well as irrational parts, of this equation are equal,

$\therefore a = x^3+3xy$, $\sqrt{b} = 3x^2\sqrt{y}+y\sqrt{y}$; $\sqrt[3]{(a-\sqrt{b})} = x-\sqrt{y}$.

The product of (1) and (2) is

(3) $$\sqrt[3]{(a^2-b)} = x^2-y = c,$$

if a^2-b is a perfect cube, and $= c$.

But $a = x^3+3xy = x^3+3x(x^2-c)$ from (3); and therefore $4x^3-3cx = a$, from which x may be found; and then y is known from the equation $y = x^2-c$.

II. *Find the sum of the squares of the first* n *natural numbers.*

We know that $1+2+3+..+n = \frac{n(n+1)}{2}$.

Let $S = 1^2+2^2+3^2+..+n^2$; then since

$$\begin{aligned} n^3-(n-1)^3 &= 3n^2-3n+1, \\ (n-1)^3-(n-2)^3 &= 3(n-1)^2-3(n-1)+1, \\ (n-2)^3-(n-3) &= 3(n-2)^2-3(n-2)+1, \end{aligned}$$

and so on to

$$\begin{aligned} 3^3-2^3 &= 3\,.\,3^2-3\,.\,3+1, \\ 2^3-1^3 &= 3\,.\,2^2-3\,.\,2+1, \\ 1^3-0^3 &= 3\,.\,1^2-3\,.\,1+1, \end{aligned}$$

we get, by adding vertically,

$$n^3 = 3(1^2+2^2+3^2+..+n^2)-3(1+2+3+..+n)+n$$
$$= 3s-\frac{3n(n+1)}{2}+n. \quad \therefore s = \frac{n(n+1)(2n+1)}{2\,.\,2}.$$

III. *Find the sum of the cubes of the first* n *natural numbers.* We have

$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1,$$
$$(n-1)^4 - (n-2)^4 = 4(n-1)^3 - 6(n-1)^2 + 4(n-1) - 1,$$
$$(n-2)^4 - (n-3)^4 = 4(n-2)^3 - 6(n-2)^2 + 4(n-2) - 1,$$

and so on to

$$3^4 - 2^4 = 4 \cdot 3^3 - 6 \cdot 3^2 + 4 \cdot 3 - 1,$$
$$2^4 - 1^4 = 4 \cdot 2^3 - 6 \cdot 2^2 + 4 \cdot 2 - 1,$$
$$1^4 - 0^4 = 4 \cdot 1^3 - 6 \cdot 1^2 + 4 \cdot 1 - 1,$$

and, by adding vertically, we get

$$n^4 = 4(1^3 + 2^3 + 3^3 + \ldots n^3) - 6(1^2 + 2^2 + 3^2 + \ldots n^2) + 4(1 + 2 + 3 + \ldots n) - n.$$
$$= 4s - \frac{6n(n+1)(2n+1)}{2 \cdot 3} + \frac{4n(n+1}{2} - n.$$
$$\therefore 4s = n^4 + 2n^3 + n^2, \quad \text{or} \quad s = \left(\frac{n(n+1)}{2}\right)^2.$$

It follows, therefore, that

$$(1 + 2 + 3 + \ldots + n)^2 = 1^3 + 2^3 + 3^3 + \ldots + n^3.$$

The sums of the fourth, fifth, &c. powers of the first n natural numbers may, in succession, be found by the same process.

IV. *To find the value of a recurring decimal.* Let P denote the p figures which do not recur, Q the q figures which do recur, and s the value of the recurring decimal; then

$$s = .PQQQ\ldots, \quad 10^p s = P.QQQ\ldots, \quad 10^{p+q} s = PQ.QQ\ldots$$

and by subtraction $(10^{p+q} - 10^p)\, s = PQ - P$. $\therefore s = \frac{PQ - P}{10^{p+q} - 10^p}$. Now, $10^{p+q} - 10^p = 10^p(10^q - 1)$; and $10^q - 1$, when expressed by figures, in the usual way, will consist of q *nines.*

Hence we find the usual rule for finding the value of a recurring decimal. *Subtract the integral number consisting of the non-recurring figures*

from the integral number consisting of the non-recurring and recurring figures, and divide by a number consisting of as many nines as there are recurring figures followed by as many ciphers as there are non-recurring figures.

V. *Demonstration of the Binomial Theorem for any exponent,* supposing the development for positive integer exponents to be known.

Suppose m and n to be positive integers; then

$$(1)\ (1+x)^m = f(m) = 1 + mx + \frac{m(m-1)}{1.2}x^2 + \frac{m(m-1)(m-2)}{1.2.3}x^3 + \&c.$$

$$(2)\ (1+x)^n = f(n) = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + \&c.$$

using the notation $f(m)$ and $f(n)$ for brevity. The product of (1) and (2) gives

$$(1+x)^m (1+x)^n = (1+x)^{m+n} = f(m) f(n).$$

But, since $m+n$ is a whole number, the development of $(1+x)^{m+n}$ must be of the same form as (1) or (2).

$$(3) \qquad \therefore (1+x)^{m+n} = f(m+n) = f(m) \times f(n).$$

For n write p, and then change m to $m+n$; then

$$f(m+n+p) = f(m+n) f(p) = f(m) \times f(n) \times f(p).$$

Continuing this process, we shall obtain

$$(4)\quad f(m+n+p+q..) = f(m) \times f(n) \times f(p) \times f(q) \times \ldots$$

Now, let $m = n = p = q = .. = \frac{s}{r}$, a fraction, s and r being positive integers, and suppose the number of fractions to be r; then (4) becomes

$$f(s) = \left(f\left(\tfrac{s}{r}\right)\right)^r. \qquad \therefore (f(s))^{\frac{1}{r}} = f\left(\tfrac{s}{r}\right).$$

But since s is a positive integer, $f(s) = (1+x)^s$, and therefore

$$\left(f(s)\right)^{\frac{1}{r}} = (1+x)^{\frac{s}{r}} = f\left(\tfrac{s}{r}\right) = 1 + \tfrac{s}{r}x + \frac{\frac{s}{r}\left(\frac{s}{r}-1\right)}{1.2}x^2 + \&c.$$

when the exponent is any positive fraction. Again, in (3) put $-n$ for n; then

$$f(-n) \times f(n) = f(-n+n) = f(0) = 1,$$

as is evident from (1) or (2).

Wherefore $\frac{1}{f(n)} = f(-n)$. But for any positive value of n we have $f(n) = (1+x)^n$.

$$\therefore \frac{1}{f(n)} = \frac{1}{(1+x)^n} = (1+x)^{-n} = f(-n)$$
$$= 1 + (-n)x + \frac{(-n)(-n-1)}{1.2}x^2 + \&c.,$$

which shows the form of the development to be the same for negative exponents. This demonstration for fractional and negative exponents is due to EULER.

EXPOSITION OF THE PROCESS OF MATHEMATICAL DEVELOPMENT.

By JOHN PATERSON, A. M., Albany, N. Y.

[Continued from Page 294, Vol. II.]

6. DURING the time $1'_t$, while the generating force φ^{n-2} of the third rank has increased duplicately from 0 to the ultimate value $3\varphi^{n-2}$, its generated force of the fourth (descending) rank, φ^{n-3}, has increased in triplicate ratio; so that if $3\varphi^{n-2}$ has generated $1\varphi^{n-3}$ in the time $1'_t$, it will generate $3\varphi^{n-3}$ in the time $1''_t$. Applying this relation to the semi-units $\frac{1}{2}'.1'_t$ and $\frac{1}{2}''.1'_t$, instead of the units $1'_t$ and $1''_t$ of time, and recollecting that since φ^{n-3} increases in triplicate ratio during the time of its genesis, its effect in an equal time with its ultimate value constant must necessarily be m'' times its effect, while increasing triplicately from zero, it is plain that the following equations are authorized.

$$\tfrac{1}{2}'.1_t' + \tfrac{1}{2}''.1_t' = 1_t':$$

$$\tfrac{1}{8}\varphi_1^{n-3} + \tfrac{3}{8}\varphi_1^{n-3} + \tfrac{3}{8}\varphi_2^{n-3} + \tfrac{1}{8}\varphi_2^{n-3} = 1\,\varphi^{n-3};$$

$$\frac{m''}{8}\varphi_1^{n-3} + \frac{3\,m''}{8}\varphi_1^{n-3} + \frac{3\,m''}{8}\varphi_2^{n-3} = \frac{m''}{8}\varphi_2^{n-3} = m''\,\varphi^{n-3};$$

the first and last terms having the relation of constant to variable, as before.

The force $\frac{m''}{8}\varphi_1^{n-3}$ would generate $\frac{m''}{8}\varphi^{n-4}$ in a unit of time, and therefore does generate $\frac{m''}{16}\varphi^{n-4}$ in the time $\frac{1}{2}''.1_t'$, the m''th part of which is $\frac{1}{16}\varphi^{n-4}$ the amount it has generated in the time $\frac{1}{2}'.1_t'$ of its own genesis, and is also the amount generated by $\frac{m''}{8}\varphi_2^{n-3}$ in the time $\frac{1}{2}'.1_t'$ of its genesis.

The force $\frac{3\,m''}{8}\varphi_1^{n-3}$ would generate $\frac{3\,m''}{8}\varphi^{n-4}$ in a unit of time, or $\frac{3\,m''}{16}\varphi^{n-4}$ in a semi-unit of time; but as it is of the third rank from its constant generator $\frac{2}{2}\varphi_1^{m-1}$, it increases in duplicate ratio, and therefore its successive effect must be divided by 3 to get its synchronous measure $\frac{3}{3}.\frac{m''}{16}\varphi^{n-4}$ for the time $\frac{1}{2}''.1_t'$.

The force $\frac{3\,m''}{8}\varphi_2^{n-3}$ would generate $\frac{3\,m''}{8}\varphi^{n-4}$ in a unit of time, or $\frac{3\,m''}{16}\varphi^{n-4}$ in a semi-unit of time; but as it is of the second rank from its constant generator $\frac{3}{4}\varphi_1^{n-2}$, it increases in simple ratio, and therefore its successive effect must be divided by 2 to get $\frac{3}{2}\frac{m''}{16}\varphi^{n-4}$, its synchronous effect in the time $\frac{1}{2}''.1_t'$. The collected terms fetch the equation $\left(\frac{1}{16} + \frac{3}{3}\frac{m''}{16} + \frac{3}{2}\frac{m''}{16} + \frac{1}{1}\frac{m''}{16} + \frac{1}{16}\right)\varphi^{n-4} = 1\,\varphi^{n-4}$, which gives $m'' = 4$,

$$\tfrac{1}{2}'.1_t' + \tfrac{1}{2}''.1_t' = 1_t':$$

and converts the last preceding equation into

$$\tfrac{4}{8}\varphi_1^{n-3} + 3.\tfrac{4}{8}\varphi_1^{n-3} + 3.\tfrac{4}{8}\varphi_2^{n-3} + \tfrac{4}{8}\varphi_2^{n-3} = 4\,\varphi^{n-3}.$$

Thus the successive and synchronous measures of a force of the fourth rank are mutually convertible by the factor 4.

NOTE. — A quantity may increase from zero by a regular law to any assigned magnitude in a given time; and it is seen by the preceding deductions that the law of increase determines the divisor for reducing successive to synchronous measure, independently of the absolute magnitude of the former. In the last preceding case, the successive semi-unit measure of the effect of the force $3 . \frac{4}{8} \varphi_2^{n-3}$ is $3 . \frac{4}{16} \varphi^{n-4}$; but as this effect increases in duplicate ratio, its synchronous measure calls for the divisor 2; while in the same place the successive semi-unit effect of $3 . \frac{4}{8} \varphi_1^{n-3}$ is also $3 . \frac{4}{16} \varphi^{n-4}$, but increases in triplicate ratio, and is reduced to synchronous measure by the divisor 3.

7. The generator $4 \varphi^{n-3}$ increases in triplicate ratio, and therefore its generatant φ^{n-4} increases in quadruplicate ratio: the successive effect of this generatant, in its function as generator, will necessarily be m''' times its synchronous effect; which authorizes the multiplication of the first of the following equations by m'''.

$$\tfrac{1}{2}' . 1_t' + \tfrac{1}{2}'' . 1_t' = 1_t' :$$

$$\tfrac{1}{16} \varphi_1^{n-4} + \tfrac{4}{16} \varphi_1^{n-4} + \tfrac{6}{16} \varphi^{n-4} + \tfrac{4}{16} \varphi_2^{n-4} + \tfrac{1}{16} \varphi_2^{n-4} = 1 \varphi^{n-4};$$

$$\frac{m'''}{16} \varphi_1^{n-4} + \frac{4 m'''}{16} \varphi_1^{n-4} + \frac{6 m'''}{16} \varphi^{n-4} + \frac{4 m'''}{16} \varphi_2^{n-4} + \frac{m'''}{16} \varphi_2^{n-4} = m''' \varphi^{n-4}.$$

The force $\frac{m'''}{16} \varphi_1^{n-4}$ increases in quadruplicate ratio from 0, being of the fifth rank from its constant generator $1 \varphi^n$: its successive unit measure of effect becomes $\frac{m'''}{32} \varphi^{n-5}$ for the semi-unit of time $\frac{1}{2}'' . 1_t'$, leaving $\frac{1}{32} \varphi^{n-5}$ for its synchronous measure in the time $\frac{1}{2}' . 1_t'$, which is also the synchronous measure of the effect of $\frac{m'''}{16} \varphi_2^{n-4}$ for the time $\frac{1}{2}'' . 1_t'$.

The force $\frac{4 m'''}{16} \varphi_1^{n-4}$ increases in triplicate ratio, being of the fourth rank from its constant generator $\frac{2}{2} \varphi_1^{n-1}$: its successive semi-unit of

effect $\frac{4\,m'''}{32}\,\varphi^{n-5}$ increases quadruplicately, and must be divided by 4 to get $\frac{4}{4}\cdot\frac{m'''}{32}\,\varphi^{n-5}$ its synchronous effect for the time $\frac{1}{2}''.\,1_t'$.

The force $\frac{6\,m'''}{16}\,\varphi^{n-4}$ increases in duplicate ratio, being of the third rank from the constant generator $\frac{3}{4}\,\varphi_1^{n-2}$: its successive semi-unit effect $\frac{6\,m'''}{32}\,\varphi^{n-5}$ increases triplicately, and must be divided by 3 to get $\frac{6}{3}\cdot\frac{m'''}{32}\,\varphi^{n-5}$ its sychronous effect for the time $\frac{1}{2}''.\,1_t'$.

The force $\frac{4\,m'''}{16}\,\varphi_2^{n-4}$ increases in simple ratio, being of the second rank from the constant generator $\frac{4}{8}\,\varphi_1^{n-3}$: its successive semi-unit of effect $\frac{4\,m'''}{32}\,\varphi^{n-5}$ increases duplicately, and must be divided by 2 to get $\frac{4}{2}\cdot\frac{m'''}{32}\,\varphi^{n-5}$ its synchronous measure for the time $\frac{1}{2}''.\,1_t$.

The collected results constitute the equation

$$\left(\tfrac{1}{3\,2}+\tfrac{4}{4}\cdot\frac{m'''}{32}+\tfrac{6}{3}\cdot\frac{m'''}{32}+\tfrac{4}{2}\cdot\frac{m'''}{32}+\tfrac{1}{1}\cdot\frac{m'''}{32}+\tfrac{1}{3\,2}\right)\varphi^{n-5}=m'''\,\varphi^{n-5},$$

which gives $m''' = 5$, and converts the last preceding equation into

$$\tfrac{1}{2}'.\,1_t'+\tfrac{1}{2}''.\,1_t'=1_t':$$

$$\tfrac{5}{1\,6}\,\varphi_1^{n-4}+\tfrac{2\,0}{1\,6}\,\varphi_1^{n-4}+\tfrac{3\,0}{1\,6}\,\varphi^{n-4}+\tfrac{2\,0}{1\,6}\,\varphi_2^{n-4}+\tfrac{5}{1\,6}\,\varphi_2^{n-4}=5\,\varphi^{n-4}.$$

Thus the successive and synchronous measures of a force of the fifth rank are mutually convertible by the factor 5.

8. So much may suffice to elucidate the genetical process occurring during the first unit of time; which process may be prolonged downwards to the nth rank, terminating with $\varpi^{n-n} = \varphi^0$, the force of the order zero which is no generator.

9. For $n = 4$, the two series of synchronous and successive unit measures of the generated forces, including the primitive generator, are

$$1\,\varphi^{\mathrm{IV}},\ 1\,\varphi''',\ 1\,\varphi'',\ 1\,\varphi',\ 1\,\varphi^0,$$

and

$$1\,\varphi^{\mathrm{IV}},\ 2\,\varphi''',\ 3\,\varphi'',\ 4\,\varphi',\ 1\,\varphi^0;$$

each term of the first series being the measure of the generated effect confined throughout to the first unit of time, while the coefficient of each term of the second series expresses the relation between the effects of its efficient in the times $1_t''$ and 1_t. Thus, while the synchronous effect of the force $3\,\varphi''$ for the time 1_t is $1\,\varphi'$ referred to synchronous effect of the latter force, it is $4\,\varphi'$ in successive measure, and the successive effect of the same force $3\,\varphi''$ is $3\,.\,4\,\varphi'$ for the time $1_t''$.

The genetical operations for the first two units of time are as follows : —

$1_t'$.	$1_t''$.
$1\,\varphi^{IV} = 1\,\varphi^{IV}$:	$\frac{1.2}{1}\varphi'''$, $\frac{1.2.3}{1.2}\varphi''$, $\frac{1.2.3.4}{1.2.3}\varphi'$, $\frac{1.2.3.4.1}{1.2.3.4}\varphi^0 = 1\,\varphi^0$.
$1\,\varphi''' = 2\,\varphi'''$:	$\frac{2.3}{1}\varphi''$, $\frac{2.3.4}{1.2}\varphi'$, $\frac{2.3.4.1}{1.2.3}\varphi^0 = 4\,\varphi^0$.
$1\,\varphi'' = 3\,\varphi''$:	$\frac{3.4}{1}\varphi'$, $\frac{3.4.1}{1.2}\varphi^0 = 6\,\varphi^0$.
$1\,\varphi' = 4\,\varphi'$:	$\frac{4.1}{1}\varphi^0 = 4\,\varphi^0$.
$1\,\varphi^0 = 1\,\varphi^0$.	

The final effect of the force $1\,\varphi^{IV}$ in two units of time is

$$(1 + 4 + 6 + 4 + 1)\,\varphi^0 = 16\,\varphi^0 = 2^4\,\varphi^0.$$

In the first left-hand column, each term below the first expresses the synchronous value of the effect generated by the immediately superior term of the second column (directly on the right of the equality signs), during the time $1_t'$; each term of the second column increasing from 0 to the value expressed by its coefficient, and being at the same time the successive value of the effect generated by its immediately superior term of the same column.

Passing to the right of the colon points, the primitive force $1\,\varphi^{IV}$ generates in the second unit of time $1_t''$, a series equal to that of $1_t'$, closing with $\frac{1.2.3.4.1}{1.2.3.4}\varphi^0 = 1\,\varphi^0$.

The force $2\,\varphi'''$, now become a constant generator, having generated $3\,\varphi''$ while increasing uniformly during $1_t'$, will generate $2\,.\,3\,\varphi''$ in the time $1_t''$: this force $2\,.\,3\,\varphi''$ increases uniformly during $1_t''$, and therefore its successive measure $2\,.\,3\,.\,4\,\varphi'$ must be divided by 2 to get its synchronous value $\frac{2\,.\,3\,.\,4}{1\,.\,2}\,\varphi'$; and since the force $\frac{2\,.\,3\,.\,4}{1\,.\,2}\,\varphi'$ increases duplicately, and is of the first order, its successive effect, $\frac{2\,.\,3\,.\,4\,.\,1}{1\,.\,2}\,\varphi^0$, has no successive value, but must nevertheless be divided by 3 to get its synchronous value, $\frac{2\,.\,3\,.\,4\,.\,1}{1\,.\,2\,.\,3}\,\varphi^0 = 4\,\varphi^0$.

The force $3\,\varphi''$, now become a constant generator, and having generated $4\,\varphi'$ while increasing duplicately during $1_t'$, will generate $3\,.\,4\,\varphi'$ in the time $1_t''$: this force $3\,.\,4\,\varphi'$ increases uniformly during $1_t''$; and as it is of the first order, its successive effect, $3\,.\,4\,.\,1\,\varphi^0$, has no successive value, but must be divided by 2 to fetch its synchronous value $\frac{3\,.\,4\,.\,1}{1\,.\,2}\,\varphi^0 = 6\,\varphi^0$.

Finally, the force $4\,\varphi'$, now become a constant generator, and being of the first order, its successive effect, $4\,.\,1\,\varphi$, contains no element of increase, or is uniform and without successive value, and therefore has but the divisor 1 to bring its synchronous value $\frac{4\,.\,1}{1.}\,\varphi^0 = 4\,\varphi^0$ for the time $1_t''$.

10. If, then, a unit force $1\,\varphi^{IV}$ of the fourth order be applied to a unit of mass 1_μ during a unit of time 1_t, at the expiration of the first unit interval $1_t'$ the mass 1_μ has described a unit of distance $1_l = 1\,\varphi^0$ the unit force of the order zero, the phenomenal measure of the primitive force in and for the first unit of time: there is in the same time $1_t'$ also generated the velocity $4\,.\,1_\lambda$ or force $4\,\varphi'$ of the first order, which will, by uniform motion, carry the unit mass 1_μ the distance $4\,.\,1_l$ in the next unit of time $1_t''$; also the forces $2\,\varphi'''$ and $3\,\varphi''$, which each generate the velocity $12\,.\,1_\lambda$ in the time $1_t''$, but under different rates of acceleration, such as suffice to carry a unit mass respectively the distances $4\,.\,1_l$ and $6\,.\,1_l$ in the time $1_t''$ of their genesis; to these amounts add the unit of distance again gen-

erated by the primitive force $1\,\varphi^{\text{IV}}$ in the second unit of time, and there is generated the distance 1_t in the first unit of time; and in the second unit of time, the distance $4\,.\,1_t$ by the velocity in existence at the beginning of this interval, and $11\,.\,1_t$ by the forces in operation; making $15\,.\,1_t$ for the time $1_t''$, and in all $16\,.\,1_t = (1+1)^4\,1_t$ for the whole time $2\,.\,1_t$.

11. As the interval of time is arbitrary, it may be taken equal to $\psi\,1_t$ or $h\,1_t$, and so achieve the transition from $(1+1)^n$ to ψ^n and $(\psi+h)^n$, etc.

As the force $1\,\varphi^{\text{IV}}$ generates $2\,\varphi'''$ uniformly in the time 1_t, it will generate $2\,\psi\,\varphi'''$ uniformly in the time $\psi\,1_t$.

As the force $2\,\varphi'''$ generates $2\,.\,3\,\varphi''$ uniformly in the time $1_t''$ and $3\,\varphi''$ duplicately in the preceding equal time $1_t'$, so the force $2\,\psi\,\varphi'''$ will generate $2\,.\,3\,\psi^2\,\varphi''$ uniformly in the time $\psi''\,1_t$ and $3\,\psi^2\,\varphi''$ duplicately in the preceding equal time $\psi'\,1_t$.

As the force $3\,\varphi''$ generates $3\,.\,4\,\varphi'$ uniformly in the time $1_t''$ and $4\,\varphi'$ triplicately in the time $1_t'$, so the force $3\,\psi^2\,\varphi''$ will generate $3\,.\,4\,\psi^3\,\varphi'$ uniformly in the time $\psi''\,1_t$ and $4\,\psi^3\,\varphi'$ triplicately in the preceding equal time $\psi'\,1_t$.

As the force $4\,\varphi'$ generates $4\,\varphi^0$ uniformly in the time $1_t''$ and $1\,\varphi^0$ quadruplicately in the time $1_t'$, so the force $4\,\psi^3\,\varphi'$ will generate $4\,\psi^4\,\varphi^0$ uniformly in the time $w''\,1_t$ and $1\,\psi^4\,\varphi^0$ quadruplicately in the preceding equal time $\psi'\,1_t$.

[To be Continued.]

Mathematical Monthly Notices.

Cambridge Senate-House Problems and Riders for the Year 1860; WITH SOLUTIONS. By the REV. H. W. WATSON, M. A., late Fellow of Trinity College; and E. J. ROUTH, M. A., Fellow and Assistant Tutor of St. Peter's College, Cambridge, and Examiner in the University of London. Cambridge: Macmillan & Co., and 23 Henrietta Street, Covent Garden, London. 1860. pp. 198. 3 plates containing 55 cuts.

The Moderators for the year 1860 are the Editors of the volume; the Examiners are PERCIVAL FROST, M. A., St. John's College, and NORMAN MACLEOD FERRERS, M. A., Gonville

and Caius College. The examples alone, collected and arranged in this series of volumes, form a valuable annual contribution to mathematical science; but their value, especially to students, is greatly increased, when, as in this volume, the problems are accompanied with solutions by those who proposed them. We earnestly invite the attention of teachers and students to this volume, not only for what it contains, but also as indicating the standard in the great mathematical University.

A Treatise on Plane Co-ordinate Geometry, as applied to the Straight Line and the Conic Sections, With numerous Examples. By I. TODHUNTER, M. A., Fellow and Assistant Tutor of St. John's College, Cambridge. Second edition revised. Cambridge: Macmillan & Co. 1858. pp. 316.

This is one of a series of eight or nine volumes, prepared by the author for the use of schools and colleges. Great care has been taken in the methods of demonstration and in the explanations to adapt them to the wants of beginners. There is no branch of mathematics in which we discover such important changes and improvements in the text-books, in use in Great Britain, as in Analytic Geometry. We have already called the attention of our readers to two valuable works upon this subject (SALMON's and PUCKLE's); and now we are able to say, that, after a careful examination, we can heartily commend the one before us. Besides the usual propositions upon the straight line and conics, it contains an exposition of many of the new processes more recently introduced into elementary works, leaving no doubt of their having secured a permanent place in elementary instruction. Each chapter is followed by a large number of appropriate examples, and at the end of the volume we find all the answers collected together, with hints for the solution of many of those which appear more difficult.

Algebra for the Use of Colleges and Schools. With numerous Examples. By I. TODHUNTER, M. A., Fellow of St. John's College, Cambridge. Second edition revised. Macmillan & Co., Cambridge, and 23 Henrietta Street, Covent Garden, London. 1860. pp. 516.

We have examined this work with unusual interest, because it is the first recent work upon the subject, used abroad, which has fallen into our hands, thus enabling us to compare it with works at home filling the same place in our system of instruction. We notice, first, that the general theory of equations is entirely omitted, and also the solution of equations of all degrees above the second; nor does the work include any discussion making it necessary to anticipate, by definition or otherwise, even the elements of the higher parts of the course; and second, the introduction of many subjects not usually treated in the Elements of Algebra. An enumeration of some of them will show the extent and direction of this innovation; a few subjects usually treated being included on account of the clearness and fulness with which they are developed. Theory of *Quadratic Expressions* in addition to Quadratic Equations; Imaginary Expressions; Ratio introductory to Proportion; Variation; Scales of Notation; Harmonical Progression; Mathematical Induction; Multinomial Theorem; Logarithms, with Exponential and Logarithmic Series; Convergency and Divergency of Series; Indeterminate Equations of the First and Second Degrees; Partial Fractions and Indeterminate Coefficients; Recurring Series; Summation of Series; Inequalities; Theory of Numbers; Probability; Miscellaneous Theorems. All these subjects are treated in a very plain and familiar style; and the whole volume possesses many of the best traits of a good text-book. The author says: "Students should be encouraged to examine carefully the language of the book they are using, so that they may ascertain its meaning, or be able to point out exactly where their difficulties arise. The language, therefore, ought to be simple and precise; and it is essential that apparent conciseness should not be gained at the expense of clearness." Students, who are pursuing their mathematical studies without the aid of a teacher, will find this work well adapted to their wants. It contains about *sixteen hundred and fifty* examples, the best collection for the schoolroom we have ever seen.

University Algebra; embracing a Logical Development of the Science, with numerous graded Examples. By CHARLES DAVIES, LL. D., Professor of Mathematics in Columbia College. New York: A. S. Barnes and Burr, 51 and 53 John Street. 1860. pp. 320.

Our acquaintance with all of the author's more advanced books, both as a student and teacher, and our conviction that they have accomplished a vast good to the cause of sound education in this country, makes us feel a deep interest in every new addition to the series. A careful examination of the work before us shows that it is intended to occupy a place intermediate between the first elements of algebra and advanced treatises, a fact which the term *University* might not indicate to all; and, that it is one of the most carefully prepared works of the author's series, possessing so many of the requisites of a good text-book, as to insure it an extensive introduction as soon as its merits shall become known. Besides the fact that the definitions, demonstrations, and explanations are all expressed with great clearness, and in that happy mean between extreme conciseness and prolixity, which constitutes a model didactic style in mathematical writing, we beg to note the following special points. The demonstration of the Binomial Theorem for positive integral powers is well adapted for beginners. A few of the successive powers of a binomial are found by actual multiplication from which the general laws of exponents and coefficients are inferred; these laws are then assumed for the mth power, and, shown to hold for the $(m+1)$st power, a species of proof known as mathematical *induction*.

The case of the Problem of the Lights discussed in the Monthly, Vol. I. pp. 160, 178, and also in the present number, is thus disposed of: "The hypothesis of $c = 0$, places the two lights at the same point, in which case they form one and the same light, whose intensity is equal to the sum of their intensities taken separately. The conditions of the problem involve the necessity of *two lights*, and the equation of the problem is found under this hypothesis. This equation ought not, therefore, to respond to the case of a single light. For the interpretation of the results obtained from an equation can only give the cases which fall under the hypothesis. The hypothesis of two lights, and the hypothesis of a single light, are not connected by any law which affords a common equation of condition. Hence, the results obtained on the supposition $c = 0$ do not belong to the problem."

It certainly is possible, mathematically, to suppose that two lights of unequal intensities may occupy the same point and still be regarded as separate and distinct lights; and the question may properly be asked whether two such lights, so situated and regarded, can illuminate any point in space by the same amount. It seems to us, therefore, that it would have been better to have shown, that, under this hypothesis, the condition that a point should be equally illuminated by the two lights becomes impossible. This would have obviated a rather doubtful explanation of the difficulty.

The chapter on the General Theory of Equations is brief, but as full as is consistent with the plan of the work. We notice that derived equations are given by simple definition, no attempt at an exposition of the principles involved being made. This seems to suggest whether it is not better to postpone the general theory of equations till later in the course, after the student shall have acquired, at least the elements of the calculus. The great fault with many of our text-books is, that they encroach upon each other's ground; and thus consume too much of the student's time by simple repetition.

After giving the form of the general equation of the nth degree, containing but one unknown quantity, with positive or negative, real or imaginary coefficients, the author says: "It is evident that this equation has at least one root, for if the first member is equal to 0, it must be so for some value of x, either real or imaginary, and this value of x is, by definition, a root." The only difficulty in this disposition of a proposition, which has lately received much attention from eminent mathematicians, is, that it assumes the point to be proved. *If a function of x equals*

zero, as a matter of course it must be so for some value of x, either real or imaginary. But is there necessarily some value of x of the form of $a+b\sqrt{-1}$ which will make the function zero? This is the question.

Astronomical Notices. — No. 21. Edited by DR. F. BRÜNNOW, Professor of Astronomy and Director of the Observatory of Michigan University, Ann Arbor. Ann Arbor, October 11th, 1860.

The contents of this number are, a letter from R. F. BOND, ESQ., on a New Clock-Escapement, with a cut; a letter from COMMANDER MAURY, announcing the discovery of a new Asteroid, September 14th, by MR. FERGUSON, with Observations of Leucothea, Comet III., 1860, and Solar Eclipse, July 18th; Co-ordinates of Spots visible on the Sun, July 18th, and Observation of the Solar Eclipse by DR. C. H. F. PETERS, Director of the Observatory of Hamilton College.

A Collection of Problems and Examples in Mathematics; selected from the Jesus College Examination Papers, with Answers. — By H. A. MORGAN, M. A., Sadlerian and Mathematical Lecturer of Jesus College, Cambridge. Cambridge: Macmillan & Co. 1858. pp. 190.

This is a collection of simple examples for the use of students, arranged as follows, with the number under each head: Euclid, 55; Conic Sections, Parabola, 27; Ellipse, 29; Hyperbola, 6; Algebra, 66; Theory of Equations, 23; Trigonometry, 96; Analytical Geometry of Two Dimensions, 94; Differential and Integral Calculus, 70; Differential Equations, 15; Geometry of Three Dimensions, 26; Elementary Statics, 61; Analytical Statics, 47; Elementary Dynamics, 57; Newton, Books I., II., III., 54; Dynamics of a Particle, 33; Rigid Dynamics, 32; Hydrostatics, 57; Optics, 72· Spherical Trigonometry, and Astronomy, 32· Answers to the Questions.

Editorial Items.

PRIZE SOLUTIONS. — Owing to the delay in the publication of the September No., Vol. II., and October No., Vol. III., one month additional will be given for the solutions of the problems in these numbers. We trust that students will continue their interest in this feature of the Monthly, and that teachers will encourage their pupils to send us solutions.

LAPLACE'S *Autograph.* — It will be seen that there is no doubt about the genuineness of this Autograph. Very little of LAPLACE'S manuscript is still in existence, and we thank MR. WATSON for enabling us to present copies of this interesting note to our readers.

MATHEMATICAL TEXT-BOOKS. — We propose to give more attention than formerly to Elementary Text-books in Mathematics, Natural Philosophy, &c. A proper proportion of each number will be thus devoted.

ERRATA. — On page 419, Vol. II., line 10, for x^2 read $n\,x^2$; p. 421, line 7, read $R=\frac{p}{n}\sqrt{1-n}$, and on line 14, read $R=\sqrt{A^2-B^2}$.

MR. EVANS has called our attention to the misstatement of Problem V. in the September Number. It should read, "Prove that a chord joining one pair of these points will meet two other such chords each at a point on an asymptote, and be *in the case of the circle* perpendicular to one of them."

Worcester's Quarto Dictionary The Standard

VERY SIGNIFICANT FACTS.

The following recommendations are from some of the most distinguished American and English scholars. They are but a few from many which have been received, testifying to the superiority of Worcester's Quarto Dictionary. These testimonials are of the highest value, for they have all been given during the present year, and after an examination of this work and of that which is endeavoring to hold the position of a rival. The scholars of America and of England, with scarcely an exception, have decided in favor of Worcester. Not a single scholar, equal in authority to any one mentioned below, can be cited as giving, after a comparison of the two works, the preference to Webster's Dictionary. We give the testimony: —

From C. C. FELTON, LL.D., *President of Harvard College.*

Aware of the labor and care which had been devoted to this (the department of scientific terms) as well as to other parts of the work, I felt assured that Worcester's Quarto Dictionary would more nearly meet the public wants than any other hitherto published.

My expectations have been more than fulfilled. I find it not only rich beyond example in its vocabulary, but carefully elaborate in all the details, and thoroughly trustworthy as a guide to the most correct and elegant usage of the language.

From the REV. JOSEPH BOSWORTH, D.D., F.R.S., *Professor of Anglo-Saxon, Oxford, England.*

It is the most complete and practical, the very best, as well as the cheapest English Dictionary that I know.

From GEORGE P. MARSH, LL.D.

The work of Dr. Worcester is unquestionably much superior to any other general dictionary of the language in every one of these particulars (orthography, pronunciation, definition, fulness of vocabulary, and precision and distinctness of definition).

From REV. W. WHEWELL, D.D., *Master of Trinity College, England.*

I have repeatedly consulted the Dictionary since it has been in my possession, and have seen reason to think it more complete and exact than its predecessors.

From CHARLES RICHARDSON, LL.D., *the oldest living English Lexicographer, England.*

I sincerely hope you may enjoy from your brethren, both in America and England, that tribute of honor to which you have earned so undoubted a title.

From D. R. GOODWIN, D.D., *President of Trinity College, Hartford.*

It was but a short time since that I was led to commend another dictionary as, on the whole, and with some exceptions, the best and most complete thing of the kind within my knowledge. The commendation was honestly given at the time; but now *it must be withdrawn in favor of yours.* I consider your dictionary, in orthography, pronunciation, and definitions, as superior to any of its predecessors.

From REV. W. B. SPRAGUE, D.D., *of Albany, N. Y.*

My opinion of Worcester's Quarto Dictionary, after having given it as extended an examination as my circumstances would admit, is, that there is no other dictionary in the language that compares with it for completeness, accuracy, comprehensiveness, and precision, and perhaps I ought to add, that I have arrived at this conclusion rather contrary to a preconceived opinion.

From REV. HENRY A. BOARDMAN, D D., *of Philadelphia.*

I particularly like it (the Dictionary), 1. Because of its very comprehensive character: 2 Because it adheres to the *settled orthography* of our noble language, — discarding those innovations which, however countenanced by certain publishing-houses, have never to any extent been accepted by the scholars of our country.

From LOUIS AGASSIZ, LL.D.

It is of great importance, when the nomenclature of science is gradually creeping into common use, that an English lexicon should embrace as much of it as is consistent with the language we speak. I am truly surprised and highly delighted to find you have succeeded far beyond my expectations in making the proper selection, and combining with it a remarkable degree of accuracy. More could hardly be given except in a scientific cyclopædia.

The following lines are quoted from Harper's Magazine for September. They serve to show very truthfully the comparative value of recent and old commendations: —

"INJUSTICE. — Our attention has been called to a species of injustice of which publishers are sometimes guilty, in publishing commendations of school-books without giving the dates when they were written. Especially does this merit reproval when these commendations are old, and when it is known that the writers have subsequently commended other and later publications in the same department. It will readily be seen that this is frequently not only an act of injustice to teachers who have had the courtesy to commend a book, but that it is also a fraud upon the public."

SWAN, BREWER, & TILESTON,

131 Washington Street, Boston.

No. 117 Washington Street, Boston, October, 1860.

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THE NORTH AMERICAN REVIEW. No. CLXXXIX. — For October, 1860.

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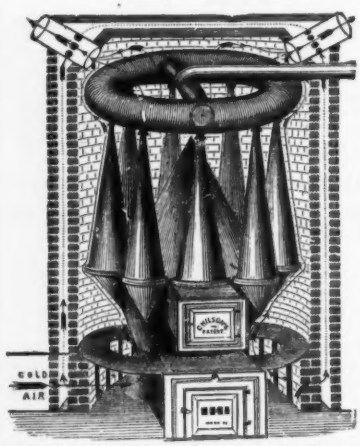

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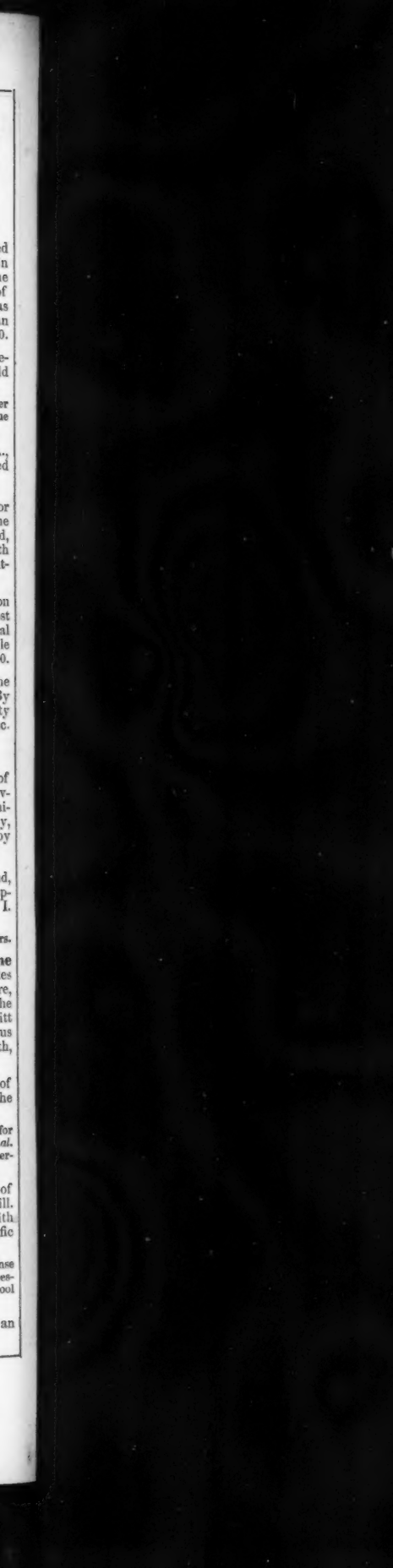

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